Efficient Finite Element Computation of Circulating Currents in Thin Parallel Strands

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Electrical machines often utilize stranded parallel conductors to reduce the skin-effect losses. This practice can lead to uneven total current distribution among the strands, increasing the resistive losses. Direct finite element analysis of circulating current problems can be computationally costly due to the large number of nodal unknowns in the conductor mesh. Methods to reduce the computational burden exist for special problems only. This paper proposes two efficient finite element formulations to solve circulating current problems with arbitrary winding configurations. According to simulations, the proposed methods yield reasonably accurate results significantly faster than the traditional brute-force approach.

***Index Terms*—Approximation methods, eddy currents, finite element analysis, proximity effects.**

I. Introduction

In random-wound electrical machines, windings are often divided into thin parallel strands to reduce the skin-effect losses. However, stranding conductors like this can lead to currents circulating between the parallel strands, occasionally almost doubling the resistive stator losses. Surprisingly little attention has been paid to finite-element (FE) analysis of these circulating currents, mainly due to the long computation times resulting from finely meshing a large number of thin strands [1]. Indeed, approaches have been mostly limited to analytical methods [2], [3]. Where FE analysis has been performed, it has focused on machines with large form-wound conductors [4].

To reduce the computation times, stranded conductors have traditionally been modelled either as a large solid conductor with a uniform equivalent current density [5], or homogenized in the frequency- or time-domain [6]–[8]. Obviously, the first approach fails to model the circulating currents at all. Similarly, practically all work on homogenization has focused purely on the skin- and proximity effects, assuming all strands to be series-connected or restricting the analysis to Litz wires [9].

This paper presents two alternative FE formulations to calculate the circulating currents in an arbitrary winding, using only a coarse mesh. The speed and accuracy of the proposed methods are evaluated on an academic test problem. According to the simulations, the methods yield reasonably accurate solutions significantly faster than the brute-force method.

II. Methods

A 2D eddy-current problem with N_s strands and N_i current paths can be described by the vector potential formulation

$$-\nabla\cdot(\nu\nabla A)+\frac{\partial}{\partial t}\sigma A-\sum_{l=1}^{N_\mathrm{s}}\frac{1}{l_\mathrm{e}}\sigma u_l=0 \tag{1}$$

$$u_l=R_l\sum_{k=1}^{N_\mathrm{i}}\eta\left(k,l\right)i_k+R_l\int\limits_{D_l}\frac{\partial}{\partial t}\sigma A\,\mathrm{d}D_l,\; l=1\ldots N_\mathrm{s} \tag{2}$$

$$U_k=R^\mathrm{ew}i_k+\sum_{l=1}^{N_\mathrm{s}}\eta\left(k,l\right)u_l,\; k=1\ldots N_\mathrm{i}\,, \tag{3}$$

where u_l are the voltages over the strands, $\eta\left(k,l\right)$ indicates if the strand l belongs to the current path k, and D_l is the domain of the strand l [4]. Using the Galerkin approach yields the following block matrix equation

$$\begin{bmatrix}\mathbf{S}^\mathrm{AA}+\mathbf{M}\frac{\partial}{\partial t} & \mathbf{S}^\mathrm{Au} & \mathbf{0}\\ \mathbf{M}^\mathrm{uA}\frac{\partial}{\partial t} & -\mathbf{I} & \mathbf{R}^\mathrm{ui}\\ \mathbf{0} & \mathbf{R}^\mathrm{iu} & \mathbf{R}^\mathrm{ii}\end{bmatrix}\begin{bmatrix}\mathbf{A}\\ \mathbf{u}\\ \mathbf{i}\end{bmatrix}=\begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ \mathbf{U}\end{bmatrix}, \tag{4}$$

with the following block elements

$$\begin{aligned}&[\mathbf{M}]_{r,c}=\int\limits_{D_l}\sigma\varphi_r\varphi_c\,\mathrm{d}D_l,\;\left[\mathbf{S}^\mathrm{Au}\right]_{r,l}=\int\limits_{D_l}-\frac{1}{l_\mathrm{e}}\sigma\varphi_r\,\mathrm{d}D_l\\ &\left[\mathbf{M}^\mathrm{uA}\right]_{l,c}=R_l\int\limits_{D_l}\sigma\varphi_c\,\mathrm{d}D_l\end{aligned} \tag{5}$$

directly related to the strand l. Shape functions are denoted by φ. A comprehensive problem description and a list of symbols will be included in the full paper.

In the brute-force approach the strands are finely meshed to obtain accurate skin- and proximity-effect losses. However, it is the authors' hypothesis that this is not necessary for obtaining reasonably accurate total currents, provided that the strands are thin compared to the skin-depth. Thus, two alternative approaches are proposed, neither of them placing any demands on the fineness or structure of the mesh used. Extension to 3D problems should be relatively simple.

1) In the *point-strand* method, the strands are assumed thin enough for the vector potential to be approximately constant over their area. With this assumption, (5) can be reduced to

$$\begin{aligned}&[\mathbf{M}]_{r,c}\approx\frac{l_\mathrm{e}}{R_l}\varphi_r(\mathbf{x}_l)\varphi_c(\mathbf{x}_l),\;\left[\mathbf{S}^\mathrm{Au}\right]_{r,l}\approx-\frac{1}{R_l}\varphi_r(\mathbf{x}_l)\\ &\left[\mathbf{M}^\mathrm{uA}\right]_{l,c}\approx l_\mathrm{e}\varphi_c(\mathbf{x}_l),\end{aligned} \tag{6}$$

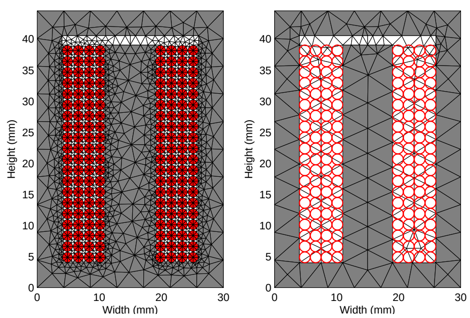
where $\mathbf{x}_l$ is the strand center.

2) In the *polygon-strand* method, D_l are approximated with polygons but are otherwise unrestricted in size or shape. For evaluating the integrals in (5) exactly, an integration scheme utilizing Gaussian quadratures and fast auxiliary triangulation is used. Details will be presented in the full paper.

III. Results and Discussion

The accuracy of the proposed methods is evaluated on a simple test problem. Uniform mesh refinement will be utilized to analyse the accuracy and computational cost evolution. Analysis is limited to linear time-harmonic problems for now.

An E-core inductor with 80 strands (1.7 mm in diameter) per slot and 4 parallel paths is analysed. Figs. 1a and 1b show the initial unrefined meshes used with the brute-force method and the proposed methods, with 1792 and 151 nodes respectively. Both meshes have 1st-order elements, and 16-gons are used in the polygon-strand method. The winding configuration is intentionally poor to obtain large circulating currents.



(a) Brute-force. (b) Proposed.

Fig. 1. Initial unrefined meshes for brute-force and proposed methods. Strands are highlighted with red.

To illustrate the potential accuracy of the proposed methods, Fig. 2 shows the behaviour of the four currents in the complex plane, as the supply frequency is increased from 10 Hz (solid dot) to 1 kHz (empty dot). The solid lines have been calculated with the brute-force method, while the dotted and dashed lines represent the point-strand and polygon-strand method, respectively. As can be seen, a good agreement between the methods has been obtained.

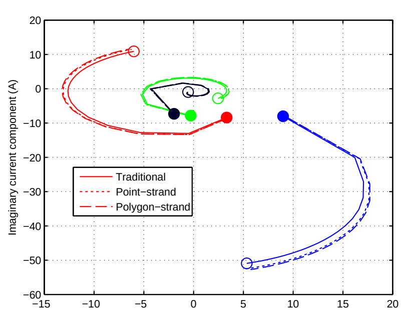


Fig. 2. Evolution of currents on the complex plane as frequency is increased.

Finally, Fig. 3 illustrates the computation times and mean errors of all three methods, obtained by repeating the simulations with different levels of *uniform* mesh refinement without any adaptiveness. Currents obtained with the brute-force method on a very dense mesh are used as reference values. Currents by the proposed methods fall within 20 % of the reference values at approximately 1/4000 of the computation cost, and within 5 % at 1/1000 of the cost. On denser meshes, the polygon-strand and brute-force method are roughly on par, while the point-strand method starts to diverge.

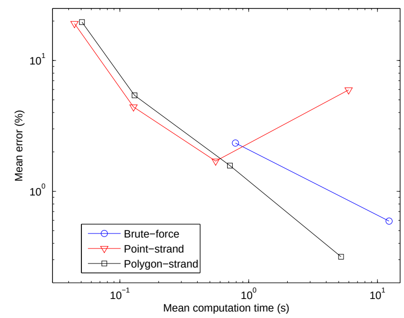


Fig. 3. Mean errors and computation times with different levels of non-adaptive mesh refinement.

A more comprehensive evaluation is underway. For now, both methods seem to yield reasonably accurate results at a fractional computational cost compared to the brute-force approach. More importantly, arbitrarily coarse meshes can be used. This is a significant improvement over the brute-force method, where meshing requirements set a high lower bound for the number of nodal unknowns.

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